

Calculation of volume with the use of “NTF” method

Izračun volumnov z uporabo “NTF” metode

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Abstract: The development of computer science, GPS, up to date instruments and methods for measurement has enabled fast collection and handling of data. Nowadays, we can come across many different programs which, on the basis of input data, give results of the shape volume. However, these programs function on the basis of the black box and we can therefore, only make assumptions about their accuracy. Due to the fact that the results, which we get with the use of programs, differ among themselves, we have made “NTF” method of volume calculation. In this article we wish to present the mentioned method and the mathematical background of the volume calculation.

Izveček: Razvoj računalništva, GPS-a, sodobnih merskih instrumentov ter metod merjenja je omogočil zelo hitro pridobivanje in obdelavo podatkov. Danes lahko srečamo celo vrsto računalniških programov, ki nam na podlagi vhodnih podatkov podajo rezultat o prostornini neke mase. Toda le ti delajo na principu črne skrinjice in o njihovi natančnosti lahko samo ugibamo. Ker se rezultati, ki jih programi podajajo med seboj razlikujejo smo zasnovali metodo izračuna volumnov »NTF«, katero vam v tem prispevku želimo predstaviti. V prispevku vam želimo prikazati, kakšna je zamisel tega načina in njegovo matematično ozadje.

Key words: volume calculation, Delauney triangulation, MS Excel, UDF

Ključne besede: izračun volumnov, Delaunayeva triangulacija, MS Excel, UDF

INTRODUCTION

The development of computer science, GPS, up to date instruments and methods for measurement has enabled fast collection and handling of data. Nowadays, we can come across many different programs which on the basis of input data give results of the shape volume. This implies to fast calculation of area or volume calculation. The way of life is also based on the fact that it is important

to acquire results in the short period of time. Due to that people nowadays use the software programs when working on calculation of volume. However, these programs function on the basis of the black box and we can therefore, only make assumptions about their accuracy.

However, the results we get with the use of programs (programs, which enable the calculation of the volume) vary among

themselves. In order to find out why these particular programs give various results we would be bound to the background of their function. Because this would be difficult we have decided to prepare the method of volume calculation in the range of our own capacities. In this way we have made “NTF” method of volume calculation. The basis of this method and the mathematical background of the volume calculation are shown in the article calculation of the volume with the use of “NTF” method.

With the consideration of mathematical principles we have used positive characteristics of program tools MS Excel and with the help of Microsoft Visual Basic for Applications¹, we have prepared User Defined Functions (UDF), which enable us to define the volume of shape in MS Excel program. The principal of volume calculation, based on Delauney triangulation, what is also the characteristic of already mentioned programs.

CALCULATION OF THE VOLUME WITH THE USE OF “NTF” METHOD

Volume of shape (dump, embankment...) can be calculated in different ways. The selection of the way, among others, depends on the shape as well as on the accuracy which we wish to reach at the volume calculation.

For the less complicated geometrical shapes, where it is possible to combine geometrical shape from regular geometrical shape, we can use way of determining the volume by abstracting volume of individual regular geometrical shape. For the volume calculation in practice the most usual principals are:

- Volume calculation from profile,
- Volume calculation from the already carried out triangulation net.

Triangle net is mostly used for modeling of the surface of the area. That is because the triangle net matches measured points on the field. The already carried out Delauney triangulation of the shape to which we wish to calculate volume with the use of “NTF” method. Delaunay triangulation is the process of calculating a triangle net that tries, among the given points, to find equilateral triangles.

The idea of the “NTF” method

If we have a series of data about the coordinate points of a particular shape, we can with the help of a suitable program from the series of data carry out Delaunay triangulation. The shape to which we wish to calculate volume, we separate with certain contour line into two parts. We get **upper** and **down** part of that shape.

¹Visual Basic for Applications (VBA) is the variety of program language Microsoft Visual Basic for programming of macro in applications for Windows operation systems

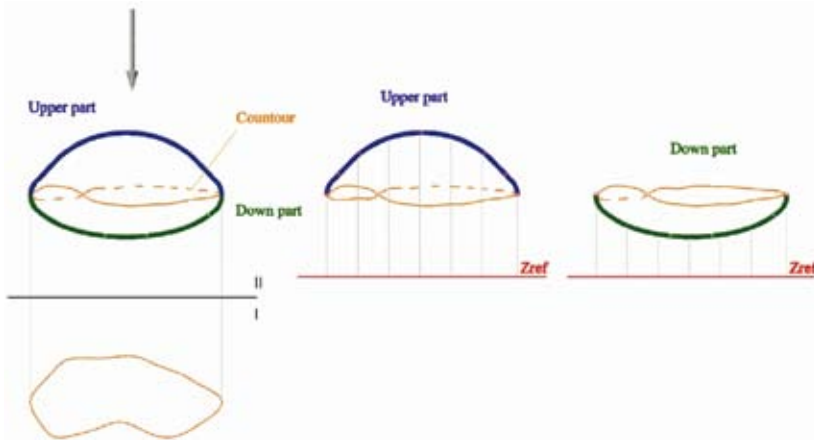


Figure 1. The separation of the shape into down and upper part
Slika 1. Razdelitev telesa na spodnji in zgornji del

To this shape with the upper and down surface we determine a certain reference high and on that reference high we create horizontal surface. In a mathematical way we can determine reference high anywhere in ending. In a numerical way it is best to choose it there where the partial amount of individual volume is the lowest, or rather where the sum of high differences of the upper and down surface equals zero. We can achieve this by determine the reference high in the centre of gravity, according to one dimension (high). But if for the reference high take minimal high from the set of the lowest location points we will not make a big mistake.

Due to the pretentiousness of determination of the centre of gravity it is recommendable to take minimal high from the set of the lowest location points. With the help of the horizontal surface on the chosen reference high and data about the edges which form triangles of the carried out Delaunay triangulation, we can solve the problem of the volume of the limited space by volume calculation of the final number of vertical triangle prism.

After we have carried out Delaney triangulation of the upper and down surface and chosen the reference high, we can on the basis of already mentioned finding calculate:

- Volume between upper surface of the shape and the surface on the reference high (volume of the upper part of the shape),
- Volume between down surface of the shape and the surface on the reference high (volume of the down part of the shape).

The difference between volumes gives us a final volume of geometrical shape.

MATHEMATICAL BASIS OF THE “NTF” METHOD

After the carried out Delaney triangulation of the surface we have the limited surface space to which we wish to calculate volume with the help of the horizontal surface on the chosen reference high and data about the edges which form triangles of the carried out Delaney triangulation, we separated on vertical triangle prism.

With this we separated, the volume of limited surface space, by volume calculation of the final number of vertical triangle prism. We are, from the mathematical point of view, interested in the volume calculation of one vertical triangle prism. An example of such prism is given in the picture below.

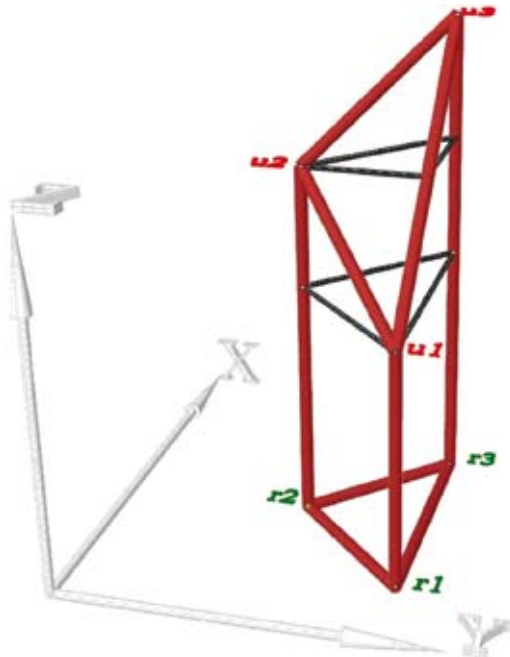


Figure 2. Example of triangle prism
Slika 2. Primer tristrane prizme

Coordinates of the given points of basis on the defined surface are:

$$\begin{aligned} u_1 &= (x_{u1}, y_{u1}, z_{ref} + h_1) \\ u_2 &= (x_{u2}, y_{u2}, z_{ref} + h_2) \\ u_3 &= (x_{u3}, y_{u2}, z_{ref} + h_3) \end{aligned} \quad (1).$$

Coordinate points of the second basis, or rather, points on horizontal surface on the reference high which is given by equitation:

$$Z = Z_{ref} \quad (2),$$

Are gained by projecting of points u_1, u_2 and u_3 on horizontal surface on the reference high. We get the coordinates of reference points that are:

$$\begin{aligned} r_1 &= (x_1, y_1, z_{ref}) \\ r_2 &= (x_2, y_2, z_{ref}) \\ r_3 &= (x_3, y_2, z_{ref}) \end{aligned} \quad (3)$$

Because of the easier volume calculation of the mentioned triangle prism, we separate this prism into three different geometrical shapes. We get two tetrahedron and one regular prism as it is shown in the picture below:

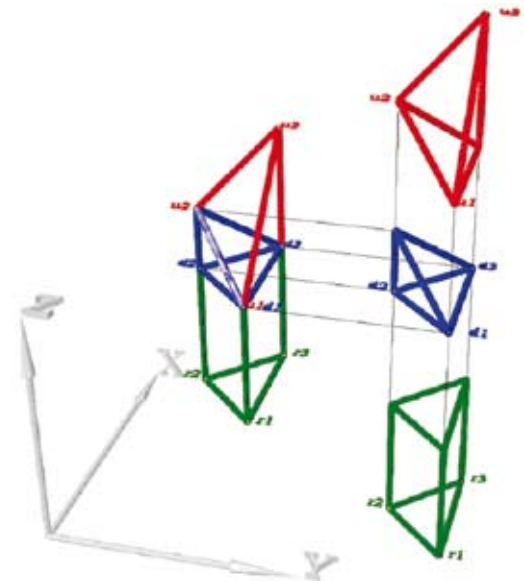


Figure 3. Separate prism
Slika 3. Razdeljena prizma

Volume calculation of separated triangle prism

Because of the better understanding and more simple way of calculation, we name the shapes after colors which are presented in Figure 3. That we get:

- Green prism (GP),
- Blue tetrahedron (BT),
- Red tetrahedron (RT).

Volume of the green prism, which is formed by edges $r_p, r_2, r_3, u_1, d_1, d_3$, we can write with a following formula:

$$V_{GP} = BASE_{norm} \cdot h_1 \quad (4),$$

At which the “ $BASE_{norm}$ ” area of normal section, therefore triangles r_p, r_2, r_3 or triangles u_p, d_p, d_3 . Formula for area of normal section is:

$$BASE_{norm} = \frac{\begin{vmatrix} x_{u1} & y_{u1} & 1 \\ x_{u2} & y_{u2} & 1 \\ x_{u3} & y_{u3} & 1 \end{vmatrix}}{2!} \quad (5).$$

Volume of blue tetrahedron which is formed by edges u_p, d_2, d_3, u_2 , we can write with:

$$V_{(BT)u_1d_2d_3u_2} = BASE_{norm} \cdot \frac{h_2 - h_1}{3} \quad (6).$$

It is known that the volume of pyramid dose not change if the top of this pyramid moves on the surface which is paralleled to basis. Due to this the volume of red tetrahedron which is formed by edges u_p, u_2, d_3, u_3 will not change if it will parallel with the surface u_p, d_3, u_3 point u_2 move into point d_2 . That is:

$$V_{(RT)u_1u_2d_3u_3} = V_{(BT)u_1d_2d_3u_2} = BASE_{norm} \cdot \frac{h_3 - h_1}{3} \quad (7).$$

If we calculate the volume of green prism, red tetrahedron and blue tetrahedron, we get final volume of mentioned triangle prism which is presented on the Figure 2. We write this as:

$$V = V_{BT} + V_{RT} + V_{GP} = BASE_{norm} \cdot \frac{h_1}{2} + BASE_{norm} \cdot \frac{h_2 - h_1}{3} + BASE_{norm} \cdot \frac{h_3 - h_1}{3}$$

$$V = BASE_{norm} \cdot \frac{h_1 + h_2 + h_3}{3} \quad (8).$$

If we now also take into consideration for the surface of normal section “ $BASE_{norm}$ ”, equation 5 we get:

$$V = \begin{vmatrix} x_{u1} & y_{u1} & 1 \\ x_{u2} & y_{u2} & 1 \\ x_{u3} & y_{u3} & 1 \end{vmatrix} \cdot \frac{h_1 + h_2 + h_3}{3!} \quad (9).$$

With the purpose of simplifying the calculation on the basis of conclusion of the above described mathematical way of volume calculation in the program MS Excel we have prepared “UDF” for volume calculation with “NTF” method.

Representation of the correct volume calculation

We carry out volume calculation with the use of NTF method in MS Excel program. For this we have, on the basis of the above mentioned findings, prepared “UDF” which enable realization of volume calculation with the use of MS Excel program.

With the purpose of examining the correctness of the volume calculation with the use of already mentioned tools. We have carried out tests in the computer program MS Excel. To this purpose we have used the prism model which is shown in the Figure 2. Data about

Table 2. Chosen reference high value

Tabela 2. Prikaz izbranih vrednosti referenčnih višine in prikaz končnega rezultata

z_{ref}	Volume UP	Volume DOWN	Accurate volume $AV=V_D-V_U$
90	4000	7000	3000
45	-500	2500	3000
40	-1000	2000	3000
35	-1500	1500	3000
20	-3000	-0	3000
15	-3500	-500	3000
10	-4000	-1000	3000
5	-4500	-1500	3000
0	-5000	-2000	3000

Table 1. Coordinate points of edge

Tabela 1. Koordinatne točke oglišč obeh trikotnikov

z_{ref}	Volume UP	Volume DOWN	Accurate volume $AV=V_D-V_U$
90	4000	7000	3000
45	-500	2500	3000
40	-1000	2000	3000
35	-1500	1500	3000
20	-3000	-0	3000
15	-3500	-500	3000
10	-4000	-1000	3000
5	-4500	-1500	3000
0	-5000	-2000	3000

coordinate points of edges of both basis are presented in the Table 1.

With the use of UDF »*point3VolumeW*« in the program MS Excel we have calculated the volume of the given prism for various reference highs. For the use of the calculation we have chosen several different values of reference high, because we wished to check if the mentioned UDF will give the same value of the volume regardless the value of reference high. Results which we have gained are presented in the Table 2.

As seen from the table above, with the use of UDF »*point3VolumeW*«, we get the same final value of the volume regardless the chosen reference high value. Due to this we can hide the reference high. In order to avoid eventual machine error it is recommended to take reference high as the smallest high from the set of given points.

In order to check whether written UDF »point3VolumeW« in the program MS Excel gives a correct result we have separated the presented prisms shape into three regular geometrical shapes in Figure 1. We calculate the volume of mentioned geometrical shapes with the use of standard mathematical forms.

$$V = V_{RT} + V_{MT} + V_{ZP} = 333,333 + 666,666 + 2000 = 3000 \quad (10)$$

From the above written result we can see that we get the same value of the volume of the prism shape as well as by the use of the above mentioned UDF »point3VolumeW«.

CONCLUSION

On the basis of the above mentioned finds we see that the final value of the volume will not change on the dependence of reference point value. No matter the value of the reference high we always get the same result. UDF also gives us the right value of the shape which we have proved in the example above. We can

with the adding up the calculated volume of the individual vertical prisms which form the volume surface limited space we calculate the final shape volume.

The way we carry out calculation on a practical example and the way of using UDF for the volume calculation with the use of NTF method will be given in the next articles.

REFERENCES

- [1] J. N. BRONŠTEJN-K. A. SEMENDJAJEV, Matematični priročnik, Tehniška založba Slovenije v Ljubljani, 1978. 683 str.
- [2] JOSEPH O'ROURKE, Computational geometry in C. Cabbidge univerity press, 1997 350 str.
- [3] VULIĆ, MILIVOJ, »User Defined Function«, februar 2005
- [4] VULIĆ, MILIVOJ, Matematične osnove določanja prostornin tristranih prizem (interna predavanja), marec 2005
- [5] VULIĆ, MILIVOJ, Osnove Delaunay-eve triangulacije in Voronoi diagramov (interna predavanja), februar 2005.