Comparative Analysis of Single Well Aquifer Test Methods on the Mill Tailing Site of Boršt Žirovski vrh, Slovenija

Primerjalna analiza metod obdelave hidravličnih poizkuso
v črpanem vodnjaku na odlagališču hidrometalurške
ejalovine Boršt, Žirovski vrh, Slovenija

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Abstract: Several aquifer tests were performed during hydrogeological investigations on mill tailings Boršt of uranium mine Žirovski vrh and large data set was generated. These results were used for comparative study of several analytical models for hydraulic test evaluation and for comparison of the results provided by them. In the present paper, methods of Jacob (COOPER & JACOB, 1967), Papadopulos et al. (1967), THEIS (1935), Hvorslev (1951) and Cooper et al. (1967) are compared. In most cases the highest values are obtained by Papadopulos method. In some cases results of Hvorslev and Cooper method are up to two and a half decades lower than results of other three methods. This is mostly due to long duration of pumping. The critical values of pumping times, where results of different slug test and pumping test methods coincide, were also defined.


Keywords: comparative analysis, slug test, pumping test, single well test

Ključne besede: primerjalna analiza, impulzni poizkus, črpalni poizkus, poizkus na črpanem vodnjaku
INTRODUCTION

During in-situ hydrogeological tests, a large number of factors affect the final outcome and only the obvious ones can be included in analytical models in order to ensure its relative simplicity. With comparative analysis one tries to assess analytical models with respect to their behavior in certain conditions, their resilience toward unexpected factor influences, and their ability to obtain representative results. Potential sources of error are usually related to local variation in permeability, leakage through pipe fittings and between pipe and adjacent soil, undetected impervious boundary close to the test, hydraulic fracturing by excessive differences in water heads, soil remolding or clogging or uprising during the test, and time lag in the piezometric responses (Chapus, 1990). In addition, errors can arise from the selection of analytical model.

During hydrogeological investigations on mill tailings Boršt of uranium mine Žirovski vrh (50 km W of Ljubljana) several pumping and slug tests were performed and large data set was generated. The characterization by hydraulic tests was made with the aim to define hydraulic permeability of the mill tailings and bedrock. These results were later used for hydrological balance calculations and groundwater numerical modeling of the mill tailings, which is positioned on large landslide.

The large data set gives us opportunity to study several analytical tests for hydraulic test evaluation and to compare the results provided by them. In the present paper, methods of Jacob (Cooper & Jacob, 1967), Papadopulos et al. (1967), Theis (1935), Hvorslev (1951) and Cooper et al. (1967) are compared.

Previous studies on the subject of comparative analysis of aquifer test methods have mostly discussed relations between different slug test methods, since they are the cheapest and therefore the most widely used methods for field evaluation of hydraulic conductivity. Herzog & Morse (1994) and Herzog (1994) compared methods of Hvorslev (1951), Cooper (1967) and Nguyen & Pinder (1984), and pointed out the importance of using several methods for calculating hydraulic conductivity, since no method can be applied at all times and employing of other methods is needed. Mace (1999) compared methods of Herbert & Kitching (1981), Barker & Herbert (1989), Hvorslev (1951), Bouwer & Rice (1976) and Cooper (1967) method for slug tests in large-diameter, hand-dug wells. He suggests that due to substantial well storage considerable pumping time may be required to lower the water level to the desired position. However, this shouldn’t affect the calculation result provided that recovery time is considerably longer than pumping time.

The relations between Hvorslev’s (1951) and Cooper’s (1967) model has been discussed by Chirlin (1989), who provides a rather assertive statement that for slug-tests the method of Cooper gives correct values of hydraulic conductivity, and Hvorslev’s result deviate from the these real values due to neglecting aquifer storativity. In contrast, Chapuis et al. (1990), Chapuis (1998) and Chapuis & Chenaf (2002) provides several independent proofs (mathematical, physical, numerical and experimental), that the theory
of Cooper et al. (1967) does not adequately represent slug-test conditions and thus can not give values of T and S.

Karanth & Prakash (1988) studied the relations between slug tests and pump tests. They indicate that transmissivity values obtained by those two methods vary mostly within a factor of three, except for pump-test transmissivity values less than 1.16 x 10^4 m^2/s.

METHODS

Construction of boreholes

On the mill tailing site Boršt and its surroundings 21 new boreholes have been installed between May and October 2003. They were organized in two groups to determine the values of hydraulic conductivity of hydrometallurgical tailings as well as the bedrock that forms the base of tailings. The depths of piezometers ranged from 5 to 25 m and from 26 to 105 m, respectively.

After the drilling, every well was rinsed with clean water and then activated with the airlift method. Activation took place until clean water flew out of the well, which was hard to achieve because in most cases dirtiness of water was a consequence of mud rinsing of hydrometallurgical tailings instead of drilling residue removal. No filter packs were installed in any of the piezometers.

Shallow piezometers are equipped with cemented ø168 mm wide surface casing and have plastic inner casing PVC-U DN 100, slot 0.75 mm. Usually this casing goes up to 3 m in the base of the aquifer, where a plug and a sink are installed as well. Deep piezometers are installed in the same manner as the shallow ones with the exception of piezometers number 3, 4 and 5 (Table 1), which have one additional inner casing and piezometer number 12, which has two additional cemented inner casings due to its greater depth.

Pumping well performance

In all 20 piezometers, hydraulic tests were performed between 4th of September and 4th of November 2003 and were interrupted by heavy rain in the mean time.

In all wells saturated thickness wasn’t big enough to create a sufficient pressure head drop to develop test correctly. In addition, a large part of the material deposited in Boršt has relatively low permeability and consequently wells have very low yield (less than 0.1 l/s). Therefore, regular pumping tests weren’t feasible in most of the wells, since the pump’s lower limit of operation is approximately at discharges 1-2 dcl/s. Improvised slug tests were performed instead. Wells were pumped at a middle rate, which was defined on the basis of previous hydrogeological interpretation. So, basic condition for slug tests, water being removed from the well nearly instantaneously, was satisfied and drawdown response wasn’t damaged. Such tests enabled the use of slug test methods as well as pumping test methods, since they satisfied all conditions for the two methods.

Prior to testing pressure probes connected to automatic data loggers were inserted in the pumping well plus in other boreholes in its vicinity. Selection of probes with sensitivities
that ranged from 1 to 3 bars depended on expected maximal drawdown (10-30 m). Time intervals between measurements were constant throughout the test but they varied between the tests from 1 to 24 seconds and were adjusted according to the duration of measurement and the expected rate of water level lowering. After a couple of tests it proved that there were no water level changes detected in the adjacent wells, therefore, we continued with testing with only one probe.

Due to absence of water level change in the wells adjacent to the pumping well we had to limit our selection of analytical methods to those that describe single-well tests (i.e. tests that don’t use any other piezometer for water level changes observation except the pumping well itself). Some of those methods were primarily developed for ordinary pumping tests with one or more piezometers (Jacob’s (1967) and Theis’s (1935) methods), but can also be used for single-well tests provided that certain additional assumptions and conditions are met.

**Analytical methods**

All methods discussed can not be applied to all performed tests in the same degree of reliance, since each method has different underlying assumption. Therefore, differences in basic conditions and assumptions that have to be satisfied for models to be successfully applied are presented in the following section as well as the governing equations.

**Models**

In this paper, methods of Jacob (Cooper & Jacob, 1967), Papadopulos (1967), Theis (1935), Hvorslev (1951) and Cooper (1967) are compared. These methods were selected while they are the most used in engineering practice. They assume boundary conditions that in most cases can reasonably be met in the field. Methods differ from one another in the given solution to the partial differential equation of groundwater flow as well as in their underlying assumptions. The later are incorporated, since field conditions in real world are way too complicated to be described with relatively simple analytical equations, therefore, certain assumptions and conditions have to be met to reduce the practical problem to mathematical constraints. Only some of them underlie all discussed methods, what leads to differences in results arising from the very fundamentals of the application.

Methods of Jacob, Papadopulos and Theis presume that (a) the well is pumped at a constant rate, whereas Hvorslev’s and Cooper’s methods suppose that (b) the water is added into or removed from the well instantaneously. In order to satisfy both conditions, improvised tests were performed where the wells were pumped at a constant rate (a) for a short time (b). If the pumping times are too long (longer than some critical time) empirical data doesn’t coincide with the slug test model. According to Mace (1999), pumping times shorter than one day do not affect the performance of slug tests. However, in case of Boršt, these critical times were proved to be much shorter, about 30 – 90 minutes.

All five methods count for the storage in the well either by using only late time data in calculations or by incorporating this consideration directly in the type curves. On the other hand, Hvorslev method assumes
incompressible water and soil, which in other words means that it does not count for storage in the aquifer as oppose to other methods discussed. As a consequence of this assumption flow toward the well is to be quasi-stationary by the Hvorslev theory, while in other methods flow is described as non-stationary. In addition, Hvorslev’s model doesn’t presume the penetration of entire aquifer and is in all less rigorous than other methods.

**Governing equations**
The general equation of groundwater flow

\[
\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t}
\]

was among the presented authors first solved by Theis (1935), who produced the following equation, derived from analogy between groundwater flow and conduction of heat:

\[
s = \frac{Q}{4\pi KD} \int_0^\infty \frac{e^{-y^2}}{y} dy = \frac{Q}{4\pi KD} W(u)
\]

where

\[
W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} ..., \text{ where } u = \frac{r^2 S}{4KDt}
\]

This equation was derived for fully penetrating pumping wells in homogeneous and isotropic non-leaky confined aquifers. An approximation \( u < 0.01 \) by Jacob is then applied to this equation, so that the terms beyond \( \ln u \) in equation (3) become so small that they can be neglected. The equation for recovery data is then rewritten as:

\[
K = \frac{2.30Q}{4\pi D\Delta s'}
\]

and

\[
t, t' > \frac{25r^2_c}{KD}
\]

where \( t \) is the time since pumping started and \( t' \) is the time since cessation of pumping, \( \Delta s' \) is change of head per one log cycle of time on semi-log plot \( s \) vs. \( t/t' \) (\( t/t' \) on the logarithmic scale).
The most frequently used method for pumping tests, namely the Jacob (Cooper & Jacob, 1946) method is also based on the presented approximation of Theis’s formula (2), except that it’s applicable to drawdown data. The later can be simplified to give the following equation for calculating hydraulic conductivity from pumping test data:

$$K = \frac{2,30Q}{4\pi D\Delta s}$$  \hspace{1cm} (6)

where $\Delta s$ is change of head per one log cycle of time on semi-log plot s vs. t (t on the logarithmic scale). An additional assumption for single well test should be satisfied:

$$t > \frac{25r_c^2}{KD}$$  \hspace{1cm} (7)

The most widely used method for slug tests has to be that of Hvorslev (1951), and is based on the following equation, derived for fully penetrating well in a confined aquifer, where a quasi-stationary flow and incompressible water and soil (i.e. zero aquifer storage) are assumed:

$$K = \frac{A}{\Delta tF} \ln \frac{h_1}{h_2}$$  \hspace{1cm} (8)

where $A$ is the cross-sectional area of the well, $l$ is the length of the tested portion, $h_1$ and $h_2$ are two values of water elevation in the well at the end and at the beginning of time interval $\Delta t$, and $F$ is the shape factor that equals:

$$F = \frac{2\pi l}{\ln\left(\frac{l}{r_w}\right)}$$  \hspace{1cm} (9)

Hvorslev presented numerous such factors for different geometries. Among those, the shape factor for cased hole and uncased or perforated extension into aquifer of finite thickness was selected and used in our calculations.

Papadopoulos et al. (1967) suggested a solution developed directly for single well pumping tests. It is based on a large-diameter wells method, where well storage cannot be neglected. The governing equation for this method is:

$$K = \frac{Q}{4\pi Ds_w} F(u_w, \alpha, \rho)$$  \hspace{1cm} (10)

where

$$F(u_w, \alpha, \rho) = \frac{8\alpha}{\pi} \int_0^\infty \frac{C(\beta)}{D(\beta)\beta^2} d\beta$$  \hspace{1cm} (11)
\[ C(\beta) = \left[ 1 - \exp\left(-\beta^2 \frac{D^2}{4u}\right) \right] \left[ J_0(\beta \rho) A(\beta) - Y_0(\beta \rho) B(\beta) \right] \]  \hspace{1cm} (12)

\[ A(\beta) = \beta Y_0(\beta) - 2\alpha Y_1(\beta) \]  \hspace{1cm} (13)

\[ B(\beta) = \beta J_0(\beta) - 2\alpha J_1(\beta) \]  \hspace{1cm} (14)

\[ D(\beta) = \left[ A(\beta) \right]^2 + \left[ B(\beta) \right]^2 \]  \hspace{1cm} (15)

\[ u_w = \frac{r_{ew}^2 S}{4KDt}, \quad \alpha = \frac{r_{ew}^2 S}{r_c^2}, \quad \text{and} \quad \rho = \frac{r}{r_w} \]  \hspace{1cm} (16)

\( K \) represents hydraulic conductivity, \( Q \) is pumping rate, \( D \) is aquifer thickness, \( s_w \) is drawdown in the well, \( r_{ew} \) is effective radius of screened part of the well, \( r_c \) is radius of the unscreened part of the well, where the water is changing, \( t \) is time since pumping started, and \( F \) is well function, which is represented by the type curves and \( J_0 \) (and \( Y_0 \)) and \( Y_1 \) are zero-order and first-order Bessel functions of the first and second kind, respectively.

Cooper et al. (1967) presented also another overlapping graphs solution for slug tests. It is based on the following equation:

\[ \frac{h_t}{h_0} = F(\alpha, \beta) \]  \hspace{1cm} (17)

where

\[ \alpha = \frac{r_{ew}^2 S}{r_c^2}, \quad \beta = \frac{KDt}{r_c^2}, \quad \text{and} \quad h_0 = \frac{V}{\pi r_c^2}. \]  \hspace{1cm} (18)

\( h_0 \) is the initial water elevation in the well, \( h_t \) is the water elevation at time \( t \), \( b \) is the dimensionless time, \( V \) is the volume of water added to the well, and \( F(\alpha, \beta) \) is the well function which is represented with a set of type curves (Figure 1, left) and is described by the equation.

\[ F(\alpha, \beta) = \frac{8\alpha}{\pi^2} \int_0^{\infty} \frac{\exp(-\beta u^2 / \alpha)}{uf(u, \alpha)} du \]  \hspace{1cm} (19)

where

\[ f(u, \alpha) = [uJ_0(u) - 2\alpha J_1(u)]^2 + [uY_0(u) - 2\alpha Y_1(u)]^2 \]  \hspace{1cm} (20)

and \( J_0(u), J_1(u), Y_0(u), Y_1(u) \) are zero-order and first-order Bessel functions of the first and second kind.

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Data processing
In general, there are two types of numerical data processing, that both base on regression principle (methods will further be addressed only by the leading author). Curve fitting methods Papadopulos and Cooper try to find best possible fit between empirical data and type curves. They read four parameters (two in each graph) that enable the calculation of hydraulic conductivity of the aquifer (Figure 1, left). Straight-line methods Jacob, Theis and Hvorslev derive the drawdown equation with certain operations and/or simplifications to the form that yields a straight-line graph (Figure 1, right). The slope of the line and its intersection with the ordinate axis facilitate the computation of aquifer parameters.

To define the subjective factor which is always present when applying different methods to aquifer test data, a short description of data segment selection criteria is given here. For drawdown data, the early time data doesn’t coincide with the Jacob model, which is a consequence of the wellbore storage. Therefore, only late time data (later than approximately 2 – 5 minutes after the start of the pumping) was used for calculation. Similarly, first part of the recovery data was excluded from the calculation as well. Results obtained from this part of recovery data are erroneous because water rises faster than normally due to water coming back from the pipe to the well after the cessation of pumping. After this water returns, the recovery data should correspond to the Theis, Hvorslev and Cooper models.

However, the subjective factor remains, since curve-matching as well as straight line matching processes were done solely by visually estimating the position of the best fit, and it was not quantified by the least-squares error function.

![Figure 1. Data processing - left: Curve-fitting Cooper method; right: Straight-line Hvorslev method (BATU, 1998)](image1)

![Slika 1. Obdelava podatkov - levo: Prilagajanje tipskim krivuljam pri Cooperjevi metodi; desno: Metoda Hvorsleva s premico (BATU; 1998)](image2)
RESULTS AND DISCUSSION

Field results

The calculation of hydraulic conductivity values from aquifer tests on Boršt site gave the following results as presented in Table 1. As can be seen from this table not all the methods could be applied to all tests. In cases number 15 and 17 to 20, slug tests with addition of water were performed, so the first three methods were inapplicable. Nevertheless, these cases were included in the analysis because of the study of the relationship between Hvorslev and Cooper method. In case number 14 the experimental data couldn’t be fitted with sufficient certainty to type curves in Papadopulos method.

Since no “real” values of hydraulic conductivity are available, we can only compare these results relatively towards one another and thus their rank numbers are given in the brackets for each test.

Ranks of the hydraulic conductivity values for several cases show some typical arrangements (Figure 3). It is evident that in almost all cases the highest or at least second highest values are obtained by Papadopulos method. Moreover, cases number 1, 3, 4, 6, 8, 11 and 16 exhibit additional similarities. Results of Hvorslev and Cooper method are up to two and a half decades lower than results of other three methods. Among those two, Hvorslev’s method results are usually somewhat higher than those of Cooper’s

Table 1. Hydraulic conductivity values and their ranks for 20 aquifer tests in Boršt, Žirovski vrh (1 – 12: deep piezometers; 13 – 20: shallow piezometers)

<table>
<thead>
<tr>
<th>Well No.</th>
<th>K_Jacob</th>
<th>K_Papadopulos</th>
<th>K_Theis</th>
<th>K_Hvorslev</th>
<th>K_Cooper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.38E-06 (1)</td>
<td>4.41E-06 (2)</td>
<td>1.84E-06 (3)</td>
<td>3.64E-07 (5)</td>
<td>4.30E-07 (4)</td>
</tr>
<tr>
<td>2</td>
<td>8.11E-07 (4)</td>
<td>1.75E-06 (2)</td>
<td>2.58E-06 (1)</td>
<td>7.06E-07 (5)</td>
<td>1.02E-06 (3)</td>
</tr>
<tr>
<td>3</td>
<td>6.46E-08 (4)</td>
<td>2.70E-07 (1)</td>
<td>2.15E-07 (2)</td>
<td>1.25E-07 (3)</td>
<td>2.58E-08 (5)</td>
</tr>
<tr>
<td>4</td>
<td>5.65E-07 (2)</td>
<td>8.11E-07 (1)</td>
<td>5.35E-07 (3)</td>
<td>1.40E-08 (4)</td>
<td>1.03E-09 (5)</td>
</tr>
<tr>
<td>5</td>
<td>6.62E-08 (5)</td>
<td>1.85E-07 (1)</td>
<td>8.28E-08 (3)</td>
<td>7.77E-08 (4)</td>
<td>1.63E-07 (2)</td>
</tr>
<tr>
<td>6</td>
<td>2.25E-06 (2)</td>
<td>3.66E-06 (1)</td>
<td>5.03E-07 (3)</td>
<td>6.51E-08 (5)</td>
<td>7.58E-08 (4)</td>
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<td>7</td>
<td>4.82E-08 (3)</td>
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<td>8</td>
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<td>3.72E-07 (1)</td>
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<td>1.31E-07 (4)</td>
<td>4.37E-08 (5)</td>
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<td>9</td>
<td>1.43E-07 (5)</td>
<td>3.74E-07 (2)</td>
<td>1.76E-07 (4)</td>
<td>2.21E-07 (3)</td>
<td>5.09E-07 (1)</td>
</tr>
<tr>
<td>10</td>
<td>4.06E-07 (4)</td>
<td>5.55E-07 (2)</td>
<td>3.40E-07 (5)</td>
<td>4.12E-07 (3)</td>
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<td>11</td>
<td>7.63E-08 (2)</td>
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<td>2.79E-08 (3)</td>
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<td>1.23E-09 (5)</td>
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<td>12</td>
<td>8.32E-07 (3)</td>
<td>2.90E-06 (1)</td>
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<td>3.69E-07 (5)</td>
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<tr>
<td>13</td>
<td>8.42E-06 (4)</td>
<td>3.01E-06 (5)</td>
<td>5.46E-05 (2)</td>
<td>1.18E-04 (1)</td>
<td>5.10E-05 (3)</td>
</tr>
<tr>
<td>14</td>
<td>4.67E-05 (1)</td>
<td>1.30E-05 (4)</td>
<td>2.27E-05 (3)</td>
<td>4.50E-05 (2)</td>
<td></td>
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<tr>
<td>15</td>
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<td></td>
<td>5.85E-07 (2)</td>
<td>7.41E-07 (1)</td>
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<tr>
<td>16</td>
<td>1.22E-05 (3)</td>
<td>5.14E-05 (1)</td>
<td>4.91E-05 (2)</td>
<td>4.02E-06 (4)</td>
<td>1.52E-06 (5)</td>
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<tr>
<td>17</td>
<td></td>
<td></td>
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<td>4.43E-05 (2)</td>
<td>7.44E-05 (1)</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>6.72E-05 (2)</td>
<td>1.45E-04 (1)</td>
</tr>
<tr>
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<td></td>
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<td>1.49E-05 (2)</td>
<td>7.09E-05 (1)</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>1.23E-07 (2)</td>
<td>1.71E-07 (1)</td>
</tr>
</tbody>
</table>

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method. Results of Theis and Jacob methods are in between Hvorslev and Cooper method results of one side and Papadopulos method values on the other. In addition, Jacob method gives a bit higher values than Theis model.

Differences between tests can also be noticed on the scatter point graphs, where pumping test method is plotted against slug test method. They are most evident in the relationship between Jacob and Hvorslev method, which are believed to be the most resilient and straightforward solutions among pumping and slug test methods, respectively. Most of the cases coincide with the equal values line, except in some cases, which fall on a line parallel to the equal values line and where results of Jacob are greater than values of Hvorslev.

Differences in results also occur between method that use drawdown data (Jacob, Papadopulos) and those that use recovery data (Theis, Hvorslev, Cooper). Nevertheless, they can only be observed where pumping times were short, otherwise they are blurred by the differences between the pumping and slug test. Overall, the recovery results show greater variability in values than those calculated from drawdown data.

![Ranges of hydraulic conductivity values](image)

**Figure 2.** Ranges of results obtained by methods of Jacob et al. (1946), Papadopulos et al. (1967), Theis (1935), Hvorslev (1951) and Cooper et al. (1967)

**Slika 2.** Razponi dobijenih vrednosti po metodah Jacoba et al. (1946), Papadopulosa et al. (1967), Theisa (1935), Hvorsleva (1951) in Cooperja et al. (1967)
In Table 2 correlations between results from different analytical models are presented. The most profound relation among the pairs of five discussed methods in case of Boršt is that of the two slug test methods – Hvorslev and Cooper. Somewhat lower are the correlations between the three pumping test methods. In general, pumping test methods do not correlate with slug test methods at such high degree.

Discussion

Results obtained by using different methods for calculation of hydraulic conductivity from aquifer tests showed a clear separation between tests with longer pumping times and those with shorter pumping timer. A term of “critical time” was introduced to distinguish between the two types of tests which describes the longest time of pumping at which slug tests methods can be employed and the results of these methods don’t diverge significantly from those of pumping test methods. Critical time was only determined approximately. MACE (1999) showed that the instant addition/removal of water condition is quite flexible and can be extended to one whole day of pumping. However, this doesn’t prove to be the case in aquifer tests on mill

Table 2. Correlations between results from different analytical models

<table>
<thead>
<tr>
<th></th>
<th>( K_{\text{Jacob}} )</th>
<th>( K_{\text{Papadopulos}} )</th>
<th>( K_{\text{Theis}} )</th>
<th>( K_{\text{Hvorslev}} )</th>
<th>( K_{\text{Cooper}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{\text{Jacob}} )</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{\text{Papadopulos}} )</td>
<td>0.92</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{\text{Theis}} )</td>
<td>0.90</td>
<td>0.86</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{\text{Hvorslev}} )</td>
<td>0.66</td>
<td>0.57</td>
<td>0.84</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( K_{\text{Cooper}} )</td>
<td>0.54</td>
<td>0.47*</td>
<td>0.68</td>
<td>0.93</td>
<td>1</td>
</tr>
</tbody>
</table>

*statistically insignificant
tailing of Boršt. Differences in results start to appear in tests with pumping times longer than half an hour, and the longer the pumping time, the greater the distinction between pumping and slug test method results. To be precise, in such cases slug test methods underestimate hydraulic conductivity values by as much as two orders of magnitude.

Differences between “pumping” and “slug” tests can be seen in typical arrangements of ranks of the hydraulic conductivity values as described in previous section. Such typical arrangement is associated largely to those cases that resemble pumping tests more than slug tests, namely cases number 1, 3, 4, 6, 8, 11 and 16. Moreover, cases of pumping times longer than “critical time” can be further distinguished from other cases as can be seen in Figure 5. “Pumping” test cases (number 1, 4, 6, 11) are the ones that diverge from the line of equal hydraulic conductivity towards the pumping test method axis, whereas “slug” test cases coincide well with this line. This shows that the results of these five methods can only be compared when a slug test was performed, and that the instant addition/removal of water condition is the most rigorous and thus the key condition to be satisfied in this matter.

Furthermore, systematic differences seem to appear between Theis’s recovery method for pumping tests and Hvorslev’s method for slug-tests. Since both are straight-line methods a clear separation between straight and non-straight line portion of the recovery data can be defined (i.e. the point where experimental data start to concur with the model) in data processing procedure. It is apparent, that in cases when our improvised aquifer test resembles slug test in higher degree than pumping test, the experimental data coincides with Hvorslev’s model earlier than with Theis’s model. On the other hand, field data falls in Theis’s model earlier, in cases with longer pumping times when performed test resembles pumping test.

As a remark, Karanth & Prakash (1988) showed that with decreasing values of hydraulic conductivity the transmissivity values of slug tests exceed values of pumping test. Since similar trends, but less extent, have been noted in this study using different methods on the same set of aquifer test data, we can conclude that a part of the slug-pumping test relationship arises solely from model structure.

Discrepancies between method that use drawdown data (Jacob, Papadopulos) and those that use recovery data (Theis, Hvorslev, Cooper) are a consequence of a couple technical facts. Firstly, not fully developed wells where activation of the well took place during the pumping resulted in lower values from drawdown data, and secondly, presence of well clogging with the fine-grained hydrometallurgical tailing which resulted in lower values from recovery data. Although these differences are small compared to the pumping/slug test differences, they add to the importance of model selection. The greater variability in results from recovery data also implies that well development is in fact an important factor.

Somewhat higher results can also be obtained by Papadopulos method as can be seen in Table 1. This is due to many cases when experimental values coincide with the straight portion of the type curves, where ambiguous results are usually gathered. This
portion of type curves represent the water being pumped from wellbore storage and is employed in cases of small well diameter and/or short pumping times as it was the case in Boršt. Nothing similar can be observed with the other curve-fitting method – Jacob method doesn’t account for the wellbore storage and thus doesn’t have such straight portions of type curves and the experimental and theoretic graphs can only be adjusted toward one another on x-axis.

Since results of Hvorslev’s method are as rule lower than those of any other method a reason for this was sought as well. Lower values could be a consequence of the difference between pumping and slug test conditions with respect to Jacob’s, Papadopulos’s and Theis’s method on one hand, and the fact that it does not account for the aquifer storage with respect to Cooper’s model on the other. Namely, after the depression due to pumping is formed, the Hvorslev model assumes that the water must fill the gaps between the soil particles in the drained portion of the aquifer, whereas storage accounting Cooper model requires additional water to compensate for the aquifer storage. Consequently, water in Cooper’s model should flow to the well slower than in Hvorslev’s model, resulting in higher values of hydraulic conductivity using the Cooper’s model given that the actual flow to the well used for calculation is in fact unique and therefore the same for both cases.

In addition, the difference between the Cooper and Hvorslev results in case of Boršt varies with the order of magnitude of the hydraulic conductivity. Since those methods could be applied to most performed tests and thus produced more results, we get a more thorough insight in their correlation. Figure 6 shows that in more permeable materials, results of Cooper exceed those of Hvorslev,

![Figure 4](image)

**Figure 4.** Relation between hydraulic conductivity values of Hvorslev and Cooper method

**Slika 4.** Razmerje med koeficienti prepustnosti po metodi Hvorsleva in Cooperjevi metodi
whereas in low permeability sediments Hvorslev’s model gives higher values. The similar phenomenon was noted by HYDER & BUTLER (1995), who noted that results of Bouwer and Rice, which can at this point be addressed as adequate to the Hvorslev method, overestimate hydraulic conductivity in clay-rich formations. The best agreement between the two models can be found in the interval $1 \times 10^{-7}$ and $1 \times 10^{-6}$ m/s. This confirms the systematic difference between the two methods. Hvorslev’s method is quasi 3-dimensional because of incorporation of shape factor, while Cooper’s method is 2-dimensional. For coarse-grained sediments one must therefore estimate the importance of vertical flow to select the correct analysis method (HERZOG, 1994).

**Conclusions**

The comparative analysis of single well aquifer test methods in Boršt Žirovski vrh showed, that methods of JACOB ET AL. (1946), PAPADOPULOS ET AL. (1967) and THEIS (1935) for pumping tests give significantly different results than methods of HVORSLAV (1951) and COOPER ET AL. (1967) in cases when aquifer tests resembled pumping tests in higher degree than slug tests. Values were comparable in cases when times of pumping were shorter than “critical times”, which were ascertained at about 30 minutes in case of Boršt. That shows that time of pumping is the key factor in this matter and that this is the most rigorous condition in case of Boršt, which is also the most easily removable one from technical point of view.

The present comparative analysis shows that the question of determination of hydraulic conductivity isn’t trivial in the interpretative sense, because the final result highly depends on the analytical model selection. Despite unique properties of existing aquifers, differences between models lead to ambiguous result, which implies that thorough knowledge of models and their boundary conditions is needed in aquifer permeability characterization.

**Povzetek**

Primerjalna analiza metod obdelave hidravličnih poizkusov v črpnam vodnjaku na odlagališču hidrometalurške jalovine Boršt, Žirovski vrh, Slovenija

Med hidrogeološkimi raziskavami na odlagališču hidrometalurške jalovine Boršt Rudnika urana Žirovski vrh je bilo izvedenih več črpalnih in nalivalnih poizkusov, na podlagi katerih je bil pridobljen velik nabor podatkov. Raziskave so bile opravljene za potrebe opredelitve koeficientov prepustnosti hidrometalurške jalovine in podlage. Opazovalne vrtine so bile izvrtnane med majem in oktobrom leta 2003. Globine vrtin, ki so izvedene v hidrometalurški jalovini znašajo med 5 in 25 m, vrtine izvedene v karnijski podlagi, sestavljeni iz kordevoljskih apnencev in dolomita ter julsko-tuvalskih klastičnih kamnin, pa so globoke med 26 in 105 m.

Debelina omočenega sloja se od vrtine do vrtine spremjina. Zaradi relativno slabe prepustnosti testiranih območij, debelina omočenega dela ni bila dovolj velika, da bi lahko s črpanjem ustvarili zadostno tlačno razliko med vodonosnikom in nivojem vode v črpalni vrtini, kar bi omogočilo ustrezen potek poizkus. Ob tem sta določene prilagoditve terjali tudi slaba prepustnost in
posledično nizka izdatnost črpalnih vrtin. Vrtine smo tako na podlagi predhodne hidrogeološke interpretacije črpal z majhno količino. S tem smo dobili uporabne podatke o znižanju nivoja podzemne vode v vodnjaku kakor tudi uporabne podatke o dvigu nivoja za obdelavo z metodami za impulzne poizkuse.<br><br>Pridobljene podatke smo uporabili za primerjalno analizo več analitičnih modelov za obdelavo hidravličnih testov. Med seboj smo primerjali tiste metode, ki obravnavajo poizkuse na črpalnih vodnjakih (t.i. single-well testi). Tako smo primerjali metode Jacoba (COOPER & JACOB, 1967), PAPADOPULOSA ET AL. (1967), THEISA (1935), HVORSLEVA (1951) in COOPERJA ET AL. (1967).<br><br>Vsaka od metod ima lastne robne pogoje, kar zahteva izpolnitve določenih predpostavk. Ujemanje oz. neujemanje teoretičnih modelov z dejanskim stanjem na terenu ima za posledico razlike v rezultatih med metodami. Obravnavane metode se med seboj ločijo tudi po načinu obdelave podatkov. Tako poznamo dva postopka numerične obdelave podatkov, ki se oba naslanjata na izračun regresije – (1) s prekrivanjem teoretičnih in empiričnih podatkov ter (2) s pomočjo trendne premice.<br><br>Na podlagi obdelave smo dobili rezultate, ki jih prikazuje Tabela 1. Ker “dejanski” podatki o prepustnosti niso na voljo, smo pridobljene koeficientne prepustnosti lahko primerjali le relativno. Posamezne koeficientne prepustnosti smo razvrstili glede na ostale vrednosti, ki so bile zabeležene v isti opazovalni vrtini in jim pripisali rang. Ti kažejo, da v večini primerov dobimo najvišje koeficiente prepustnosti s Papadopulosovo metodo.<br><br>V nekaterih primerih so koeficienti prepustnosti izračunani z metodami po Hvorslevu in Cooperju za dve in pol dekadi nižji od rezultatov ostalih metod, kar je v glavnem posledica trajanja črpanja. Operdelili smo tudi t.i. “kritične čase”, torej tiste čase trajanja črpanja, ob katerih se rezultati različnih metod še razmeroma skladajo. Ti so v primeru Boršta med 30 in 90 minutami in so specifični za to območje. Za primerjavo lahko podamo primer s severnega Teksasa, ki ga navaja MACE (1999), v katerem črpalni časi, krajiš od enega dneva naj ne bi vplivali na kvaliteto impulznega poizkusa. Efekt predolgega črpanja prikazuje Slika 3, kjer so v točkastem diagramu prikazani koeficienti prepustnosti po Jacobu proti koeficientom prepustnosti Hvorsleva. Večina rezultatov (krogi) se razporedi vzdolž premice z naklonom 1 in presečiščem 0, nekatere vrednosti (kvadrati) pa padejo na premico z enakim naklonom, ki je zamaknjena proti osi Hvorsleva. S krogi so ponazarjeni tisti poizkusi, ki so po dolžini črpanja bolj podobni impulznim poizkusom, s kvadrati pa tisti poizkusi, ki so bolj podobni črpalnim poizkusom.<br><br>Primerjalna analiza med rezultati je pokazala, da je čas črpanja ključni dejavnik in da lahko primerljive rezultate na območju Boršta dobimo pri črpalnih poizkusih, ki niso daljši od 30 minut. Hkrati se je izkazalo, da vprašanje opredelitve koeficienta prepustnosti v interpretativnem smislu ni trivialno, saj je končni rezultat v veliki meri odvisen od izbrane metode obdelave. Tako kljub enoizračnemu stanju v naravi razlike med robnimi pogoji posameznih metod vodijo do dvoumihh rezultatov, zaradi česar je za pravilno karakterizacijo hidravlične prepustnosti potrebno temeljito poznавanje modelov in pogojev njihove uporabe.
REFERENCES


