River basin networks morphological analysis in scope of entropy

Levent Yilmaz

Hydraulic Division, Civil Engineering Faculty, Technical University of Istanbul, 80626 Maslak, Istanbul, Turkey; lyilmaz@itu.edu.tr

Abstract: Under the assumption that the only information available on a drainage basin is its mean elevation, the connection between entropy and potential energy is explored to analyze drainage basins morphological characteristics. Nearly, 30 years ago, Leopold and Langbein (1962) applied for the first time the concepts of physical entropy to study the behavior of streams. Their application was based on the analogy between heat energy and temperature in a thermodynamic system and potential energy and elevation, respectively, in a stream system. Two thermodynamic principles were applied. The first principle is that the most probable state of a system is the one of maximum entropy.

Introduction

The second is the principle of minimum entropy production rate. Using these principles, Yang (1971) derived for a stream system the law of average stream fall, and the law of the least rate of energy expenditure. Yang and coworkers (Song & Yang, 1990) and others have since applied the latter law to a range of problems in hydraulics. The connection between entropy and potential energy, which these workers so successfully exploited to investigate river engineering, sediment transport, and other problems, was not exploited in hydrology. In this work we pursue this connection to derive relations between entropy and mean elevation for a drainage basin network and to derive relations for the river profiles.

Much of the work employing the entropy concepts in hydrology has been with the application of informational entropy. The beginnings of such a work can be traced to Lienhard (1964), who used a statistical mechanical approach to derive a dimensionless unit hydrograph of a drainage basin.

It may be visualized that the study of the landscape is the study of constraints imposed by geologic structure, lithology, and history. The way in which some constraints affect the river profile can be evaluated if one considers the profile to approximate its maximum probable condition under a given set of constraints. The most important observations are summarized as below:

a) The absence of all constraints leads to no solution.
b) The longitudinal profile of a stream system subject only to the constraint of base level is exponential with respect to elevations above base level.
c) The profile of a stream subject only to the constraint of length is exponential with respect to stream length which is a logarithmic function with respect to elevation.

Short scientific paper

d) Introduction of the constraint of a partial base level above that of the sea adds a measure of convexity in the profile.

STREAM POWER

Although many formulas for sediment transport have been devised, most can be expressed in terms of stream power as suggested by BAGNOLD (1960). Power is an important factor in the formulation of the hydraulic geometry of river channels. As explained by Bagnold, the stream power at flows sufficiently great to be effective in shaping the river channel is directly related to the transport of sediment, whose movement is responsible for the channel morphology. LAURSEN (1958) gives several typical equations for the transport of sediment, based on flume experiments and the average relation shows sediment transport in excess of the point of incipient motion to vary about as $(vDS)^{1.5}$ where vDS is the stream power per unit area. In terms of sediment per unit discharge, that is the concentration, C, the several equations average out as $C \propto n\,(vD)^{0.5}\,S^{1.5}$, a result that is consistent with the conclusion reached by BAGNOLD (1960). There is in addition to be considered the effect of sediment size. Examination of several equations indicates that sediment transport varies inversely as about the 0.8 power of the particle size. There have been several attempts to relate particle size to the friction factor n and by using the Strickler relation that the value of n varies as the 1/6 power of the particle size. It is realized full well that both the sediment transport and the friction factor are influenced by many other factors such as bed form and the cohesiveness, sorting, and texture of the material. These are the kinds of influences, themselves effects of the river, that prevent a straightforward solution of river morphology. In order to limit the number of variables only the effect of particle size on transport will be considered, as this factor varies systematically along a river from headwater to mouth. Thus, sediment transport concentration is given as $C \propto (vD)^{0.5}\,S^{1.5} / n^4$. The sediment transport per unit discharge in the river system will be recognized as a hydrologic factor that is independent of the hydraulic geometry of a river in dynamic equilibrium. Consequently sediment concentration may be considered constant.

Thus, there are three equations: continuity, hydraulic friction, and sediment transport. There are five unknowns. The two remaining equations will be derived from a consideration of the most probable distribution of energy and total energy in the river system.

The probability of a given distribution of energy is the product of the exponential functions of the ratio of the given units to the total as

$$p \propto e^{-\frac{E_{n1}}{E}}\, e^{-\frac{E_{n2}}{E}}\, e^{-\frac{E_{n3}}{E}} \ldots etc. \qquad (1)$$

The ratios of the units of energy E_1, E_2, etc., representing the energy in successive reaches along the river sufficiently long to be statistically independent, to the total energy E in the whole length, are E_1/E ; E_2/E; …… E_n/E. The product of the exponentials of these is the probability of the particular distribution of energy. As previously, the most probable condition is when this joint probability, p, is a maximum and this exists when $E_1 = E_2 = E_3 \ldots = E_n$. Thus energy tends to be equal in each unit length of channel (LEOPOLD AND LANGBEIN, 1962).

Equable distribution of energy corresponds to a tendency toward uniformity of the hydraulic properties along a river system. Considering the internal energy distribution, uniform distribution of internal energy per unit mass is reached as the velocity and depth tend toward uniformity in the river system. Since the energy is largely expended at the bed equable distribution of energy also requires that stream power per unit of bed area tend toward uniformity. An opposite condition is indicated by PRIGOGINE's (1955) rule of minimization of entropy production which leads to the tendency that the total rate of work, $\sum QS\Delta Q$ in the system as a whole be a minimum. Because $S \propto Q^z$, then $\sum Q^{1+z}\ \Delta Q \rightarrow$ a minimum. For a given drainage basin this condition is met as z takes on increasingly large negative values. However, there is a physical limit on the value of z, because for any drainage basin the average slope $\sum S\Delta Q / \sum \Delta Q$ must remain finite. This condition is met only for values of z greater than –1, and therefore z must approach –1 or 1 + z approaches zero. The condition of minimum total work tends to make the profile concave; whereas the condition of uniform distribution of internal energy tends to straighten the profile. Hence, we seek the most probable state.

The most probable combination is the one in which the product of the probabilities of deviations from expected values is a maximum. It is unnecessary to evaluate the probability function, provided one can assume normality, as we can then state directly that the product of the separate probabilities is a maximum when their variances are equal.

$$\left(\frac{F_1}{\sigma_{F_1}}\right)^2 = \left(\frac{F_2}{\sigma_{F_2}}\right)^2 = \left(\frac{F_3}{\sigma_{F_3}}\right)^2 = etc. \qquad (2)$$

where F_1, F_2, F_3 represent the several functions. The standard deviations σ_m, σ_f, σ_z, and σ_y represent the variability of the several factors as may occur along a river system. Since these values are not known initially, the problem must be solved by iteration (LEOPOLD AND LANGBEIN, 1962). Fortunately, the solution is not sensitive to the values of the several standard deviations, so the solution converges rapidly. Therefore,

$$\left(\frac{F_1}{\sigma_{F_1}} + \frac{F_2}{\sigma_{F_2}}\right)\left(\frac{F_1}{\sigma_{F_1}} - \frac{F_2}{\sigma_{F_2}}\right) = 0 \qquad 3)$$

for which there are two possible solutions:

$$\left(\frac{F_1}{\sigma_{F_1}} + \frac{F_2}{\sigma_{F_2}}\right) = 0 \qquad (4)$$

or

$$\left(\frac{F_1}{\sigma_{F_1}} - \frac{F_2}{\sigma_{F_2}}\right) = 0 \qquad (5)$$

To summarize, we have introduced three statements on the energy distribution:

$$\begin{aligned} &\frac{1}{2}z - y \rightarrow 0 \\ &m + f + z \rightarrow 0 \\ &(1 + z) \rightarrow 0 \end{aligned} \qquad (6), (7),(8)$$

The absolute values of the standard deviations need not be known, as we can infer their relative values. For example, letting $F_1 = (1/2)z - y$, the standard deviation of F_1 is

$$\sqrt{(\sigma_{z/2})^2 + \sigma_y^2} \quad (9)$$

and $F_2 = m + f + z$; $\sigma_{F_2} = \sqrt{\sigma_m^2 + \sigma_f^2 + \sigma_z^2}$;
$F_3=(1+z)$; $\sigma_{F3} = \sigma_z$

$$F_2=m+f+z \quad (10)$$

LEOPOLD AND MADDOCK (1953) describe and evaluate from field data the hydraulic geometry of river channels by a set of relations as follows:

$$v \propto Q^m \quad (11)$$

$$D \propto Q^f \quad (12)$$

$$w \propto Q^b \quad (13)$$

$$S \propto Q^z \quad (14)$$

$$n \propto Q^y \quad (15)$$

where v is the mean velocity, D is the mean depth, w is the surface width, and S is the energy slope, and n is the friction factor at a cross section along a river channel where the mean discharge is Q. It is desired to evaluate the exponents in a downstream direction as discharge of uniform frequency increases. Some of the principles that have been described can provide estimates of the magnitude of the exponents of the above relationships. The exponents m, f, b, z, and y describe the variability in velocity, depth, width, slope, and friction along a river channel, but do not uniquely determine the magnitudes of these properties. The first condition is that specified by the equation of continuity Q=vDw, which requires that

$$m + f + b = 1.0 \quad (16)$$

The solution of their values is not available for the first trial solution so all values are considered equal.