Modelling groundwater effects on slope stability

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Abstract: The present work is aimed at validating a model for defining the relationship between precipitation and the rise of piezometric level by solving Richards equation. The limit equilibrium method allows to determine the conditions and points in which a strong risk of instability exists. Based on the model output data, a drainage system can be designed that ensures the groundwater table is maintained beneath the critical limit.

Keywords: groundwater, modelling, slope stability

Introduction

The rise in water table on a slope after precipitation events may cause slope failure.

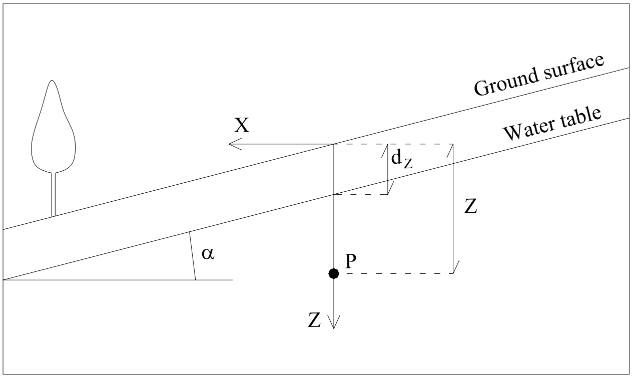

Figure 1. Definition of coordinates

According to the one-dimensional infinite-slope stability analysis (limit equilibrium method), failure occurs at depth Z if at that depth the safety factor F_s becomes equal to one. F_s results from three contributions: soil friction, cohesion and infiltration.

$$F_s = \frac{\tan\varphi}{\tan\alpha} + \frac{c}{\gamma_s Z \sin\alpha\cos\alpha} + \frac{-\psi(Z,t)\gamma_w \tan\varphi}{\gamma_s Z \sin\alpha\cos\alpha} \tag{1}$$

where: φ is the soil friction angle, α is the slope angle, c is the soil cohesion, γ_s is the depth-averaged soil unit weight, γ_w is the unit weight of groundwater, Z is the vertical depth (figure 1), ψ (Z,t) is the pressure head and t is time.

The determination of ψ (Z,t) must take into account the processes that take place in the unsaturated zone during transient and stable flow (Torres et al., 1998).

Scientific paper

THE MODEL

The model employed for this study is based on a modified version of the R. M. Iverson model (IVERSON, 2000). It requires the following input data: slope angle α; vertical depth of analysis Z; vertical steady state water table depth d_Z (figure 1); contributing area A (catchment area that potentially affects groundwater pressures at depth Z); soil friction angle φ; soil cohesion c; soil unit weight γ_s; saturated hydraulic conductivity K; saturated hydraulic diffusivity D; vertical rainfall intensity I and rainfall duration T. Iverson introduces two timescales: Z^2/D (short-term) and A/D (long-term). The square root of the ratio of the two timescales yields a length scale ratio $\varepsilon = Z/(A)^{1/2}$. Unsteady, variably saturated, Darcian flow of groundwater is governed by Richards equation (PANICONI ET AL., 1991; FREEZE & CHERRY, 1979). The solution of the equation can be expressed algebraically with five simplifying assumptions: (1) $\varepsilon \ll 1$; (2) $T \ll A/D$; (3) D varies negligibly; (4) landslide mechanics can be represented adequately by an "infinite slope" force balance; (5) soil strength depends on constant Coulomb parameters friction angle and cohesion. Long-term behaviour is characterized by steady state pressure head response that, for slope-parallel groundwater flow, can be expressed in the form:

$$\psi = (Z - d_Z)\cos^2\alpha \tag{2}$$

Short-term behaviour is described by imposing the following initial and boundary conditions: (1) at the onset of rainfall a steady state pressure head distribution exists; (2) at great depths steady state pressures persist and (3) Darcy's law governs water entry at the ground surface. For slope-parallel groundwater flow the solution of Richards equation can be written:

$$\frac{\psi}{Z}(Z, t \le T) = \left(1 - \frac{d_Z}{Z}\right)\cos^2\alpha + \frac{I}{K}\left(R^*(t)\right) \tag{3}$$

$$\frac{\psi}{Z}(Z, t > T) = \left(1 - \frac{d_Z}{Z}\right)\cos^2\alpha + \frac{I}{K}\left(R^*(t) - R(t^* - T^*)\right) \tag{4}$$

where:

$$t^* = \frac{tD'}{Z^2};\; T^* = \frac{TD'}{Z^2};\quad D' = 4\,D\cos^2\alpha \tag{5}$$

$$R(t^*) = \sqrt{\frac{t^*}{\pi}}\exp\left(-\frac{1}{t^*}\right) - erfc\left(\frac{1}{\sqrt{t^*}}\right) \tag{6}$$

D' is the effective hydraulic diffusivity and R(t*) is the pressure head response function.

Numerical solution

The conceptual model described above was implemented using a Fortran 90 numerical code that was tested using some cases reported in Iverson's work (Iverson, 2000) to verify qualitative agreement of the results. Once tested, the model was applied to a landslide event (Cortone, 1999) that occurred on a hillslope located in Tolve (Bari, Italy) in the winter of 1998 (Delitala, Soddu et al., 2002), giving satisfactory results. The analysis clearly showed that slope failure could be attributed to earth movment at the foot of the hill during construction of a road, rather than to a rise in water table.

On the contrary, in the following example a rainfall event of duration T=10 min and intensity $I=5x10^{-5}$ m/s causes a critical rise in water table and produces a landslide. The example shows how the model output can be used to design a drainage system.

The analysis was conducted at depth Z=1.4 m, with steady state water table at depth d_z=0.7 m. The slope angle α is 31° and the soil is characterized by the following parameters: φ=38°, c=500 Pa, γ_s=19000 N/m^3, K=10^{-4} m/s, D=10^{-3} m^2/s. The contributing area A is 10000 m^2.

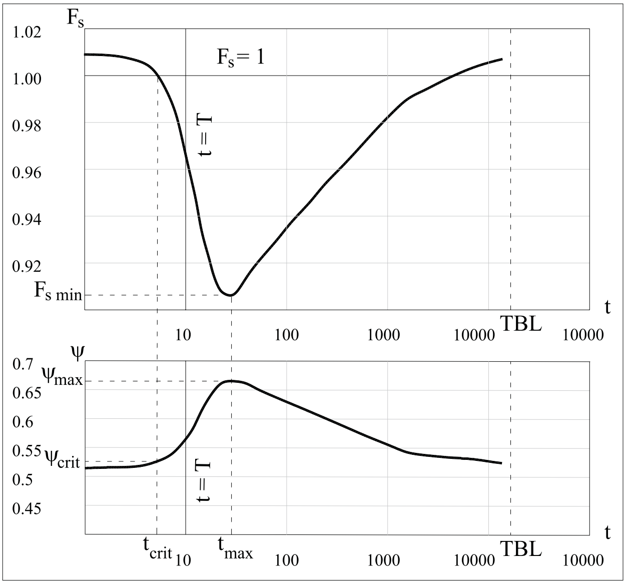

Figure 2. Pressure head responses and factor of safety predicted by the model.

In Figure 2, the safety factor F_s and the pressure head ψ are plotted against time. The 2nd diagram shows the typical shape of the function $\psi(t)$: ψ increases during rainfall (from t=0 to t=T) and, after rainfall ceases, continues to increase up to time t_{max}. The time t=TBL=16667 min marks the start of the long-term period, in which steady state conditions are re-established. But, in this case, for $t=t_{crit}$=7 min and $\psi=\psi_{crit}$=0.53 m, Fs equals

1 and the landslide occurs. So, for t> t_{crit}, the diagrams have no physical meaning. However, they do allow to calculate that, to prevent slope failure, 0.0087 m/min of water needs to flow off through a drainage system.

CONCLUSIONS

The model presented here relates the rise of groundwater table during a rainfall event to several factors associated with rainfall, soil properties and slope geomorphology, taking into account the presence of an unsaturated zone as well as the processes taking place during transient flow. The model is based on the solution of Richards equation under simplifying assumptions that still provide, however, a reasonably realistic approximation of the processes. Using the limit equilibrium method it is possible to find a critical pressure head ψ_{crit} above which slope failure takes place and to design a drainage system in order to maintain the groundwater table below ψ_{crit}.

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